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A METHODOLOGY TO DESIGN EFFICIENTLY TRANSIT SINGLE ROUTES & TIMETABLES

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1. INTRODUCTION

Transit schedulers certainly understand the need to accommodate the observed passenger demand as well as possible. However, at the same time, their effort is also directed to the minimization of vehicle and driver costs. The trade-off between increasing passenger comfort and reducing the cost of service makes the schedulers' task extremely cumbersome and complex. Therefore, there is a need to supply the schedulers computerized procedures with alternative schedule options along with interpretation and explanation of each alternative.

The first phase of this research, which has been completed and documented (1,2,3,4), provides procedures to derive alternative timetables (using passenger load data) along the entire transit route without short-turn trips. A short-turn trip is initiated beyond the route departure terminal and/or terminated before the route arrival terminal. The possibility to generate short lines opens the opportunity to further save vehicles while ensuring that the passenger load in each route segment will not exceed the desired occupancy (load factor).

In fact, the schedulers at most transit properties usually include short-turn operating strategy in their efforts to reduce the cost of service. The procedures commonly used by them are based only on visual observation of the load profile (the distribution of the loads along the entire route). That is, a potential turn point is determined at the nearest adequate time point (major stop) to a stop in which a sharp decrease or increase in the passenger load is observed. While this procedure is intuitively correct, the schedulers do not know if all the short-turn trips are actually needed to reduce the fleet size. On the other hand, each short-turn trip limits the service and hence, tends to reduce the passenger level-of-service.

One work which is related to this study is reported by Furth et al., (5), and includes an overview of operating strategies on major downtown-oriented bus routes. Among the strategies discussed are the short-turn trips where the service trip is initiated further down along the route, but the arrival point of all the trips is the same. This work designs all the possible categories of short-turn trips for any type of transit lines (crosstown routes, downtown-oriented routes, feeder routes, etc.).

The major objectives set forth in this work are:

(i) to derive the minimum fleet size required to carry on a given timetable (including the consideration of deadheading - nonrevenue trips);

(ii) to adjust the number of departures in each short-turn point to that required by the load data, provided that the maximum headway to be obtained is minimized. (This objective results in the maximum possible short-turn trips and the minimum required fleet size);

(iii) to minimize the number of short-turn trips provided that the minimum fleet size is maintained (for a given timetable, this objective results in increasing the level-of-service seen by the passengers).

In order to satisfy the objectives, several methods were developed. These methods are based on procedures and algorithms which utilize data commonly inventoried or collected by most transit properties. Furth (6) uses origin-destination (O-D) data to assess short-turn strategies for route 16 in Los Angeles (SCRTD) between West Hollywood and downtown. While the O-D data can improve the scheduling of short-turn trips, it is commonly unavailable at the transit agencies. This work is not based on O-D data, but its methods can be extended to include such data whenever it is available.

Confusing. Furth et al or current paper.

2. BACKGROUND

2.1. Deficit Function

A description follows of the deficit function approach for assigning the minimum number of vehicles to

carry on a given timetable. A deficit function is simply a step function which increases by one at the time of each trip departure and decreases by one at the time of each trip arrival. Such a function may be constructed for each terminal in a multi-terminal transit system. To construct a set of deficit functions, the only information needed is the transit timetable. The main advantage of the deficit function is its visual nature.

Let $d(k,t)$ denote the deficit for point k at time t. This point k can be either a terminal or a timepoint provided that some trips are initiated and/or terminated at this point. The value of $d(k,t)$ represents the total number of departures less the total number of trip arrivals up to and including time t. The maximal value of $d(k,t)$ over the schedule horizon is designated $D(k)$.

It is possible to partition the schedule horizon of $d(k,t)$ into a sequence of alternating hollow and maximal intervals. The maximal intervals define the interval of time over which $d(k,t)$ takes on its maximum value. A hollow interval is defined as the interval between two maximal intervals. Hollows may consist of only one point, and if this case is not on the schedule horizon boundaries, the graphical representation of $d(k,t)$ is emphasized by a clear dot.

If we denote the set of all the route and points (terminals or timepoints) as E, the sum of $D(k)$ for all $K \in E$ is equal to the minimum number of vehicles required to service the set E. This is known as the Fleet Size Formula, independently derived by Bartlett (7), Gertsbach and Gurevich (8), and Salzborn (9,10). Mathematically, for a given fixed schedule:

$$N = \sum_{k \varepsilon E} D(k) = \sum_{k \varepsilon E} \max_{t} d\ (k,t) \tag{1}$$

where N is the minimum number of vehicles to service the set E.

When deadheading (DH) trips are allowed, the fleet size may be reduced below the level described in Eq. (1). Ceder and Stern (11) describe this procedure based on the construction of a Unit Reduction Deadheading Chain (URDHC). Such a chain is comprised of a set of non-overlapping DH trips which, when inserted into the schedule, reduces the fleet size by one. The procedure continues inserting URDHC's until no more can be ins-

erted or a lower bound on the minimum fleet is reached. Determination of the lower bound is detailed in Stern and Ceder (12). The deficit function theory for transit scheduling is extended by Ceder and Stern (13,14) to include possible shifting in departure times within bounded tolerances.

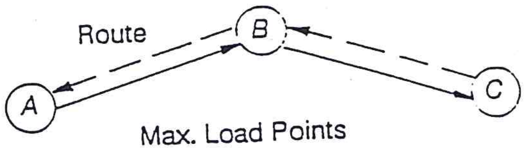
2.2. Minimum Fleet Size for a Complete Timetable: An Example

The deficit function theory described in the previous section is used to determine the minimum number of vehicles required to cover the complete timetable without short-turn trips. This minimum size is designated N_{min}.

A simple example is used as an expository device to illustrate the deficit function approach and the procedures developed. This example appears in Fig. 1. It is based on a given timetable that covers about a two-hour schedule. These hours refer to the departure times at the minimum load points. The route (set R) is comprised from three timepoints: A, B, C and the average travel times for service and deadheading trips are also given in Fig. 1.

Based on the deficit function approach, it is possible to construct d(A,t) and d(C,t). The minimum number of vehicles required without deadheading trips is D(A) + D(C) = 11. However, a DH trip can be inserted from A to C -- departing after the last maximal interval of d(A,t) and arriving just before the start of the first maximal interval of d(C,t). Both d(A,t) and d(C,t) are then changed according to the dashed-line in Fig. 1. It results in reducing D(C) from 6 to 5 and the overall fleet size from 11 to 10. After that, it is impossible to further reduce the fleet size through DH trip insertions and hence $N_{min} = 10$.

The latter observation can also be automatically detected by the lower bound test. The simple lower bound (11) is equal to the maximum value of the combined function (with respect to the time): d(A,t) + d(C,t). Following the DH trip insertion procedure, the maximum of the combined functions is 10 and therefore, N_{min} reaches its lower bound. An improved lower bound method appears in (12).



Direction	A - C			C - A			
Time-point	A	B	C	C	B	A	
7 - 8 at B	7:00	7:15	7:40	7:00	7:20	7:35	7 - 8 at C
	7:10	7:25	7:50	7:15	7:35	7:50	
	7:25	7:40	8:05	7:20	7:40	7:55	
	7:35	7:50	8:15	7:25	7:45	8:00	
	7:40	7:55	8:20	7:30	7:50	8:05	
8 - 9 at B	7:45	8:05	8:30	7:40	8:00	8:15	
	7:50	8:10	8:35	7:50	8:10	8:25	
	8:00	8:20	8:45	8:05	8:30	8:45	8 - 9 at C
	8:15	8:35	9:00	8:15	8:40	8:55	
	8:25	8:45	9:10	8:20	8:45	9:00	
	8:40	9:00	9:25	8:27	8:52	9:07	

Hours at max. load point	Travel times (min.)				*DH* times (min.)		
	A - B	B - C	C - B	B - A	A - C	A - B	B - C
7 - 8	15	25	20	15	25	15	5
8 - 9	20	25	25	15			

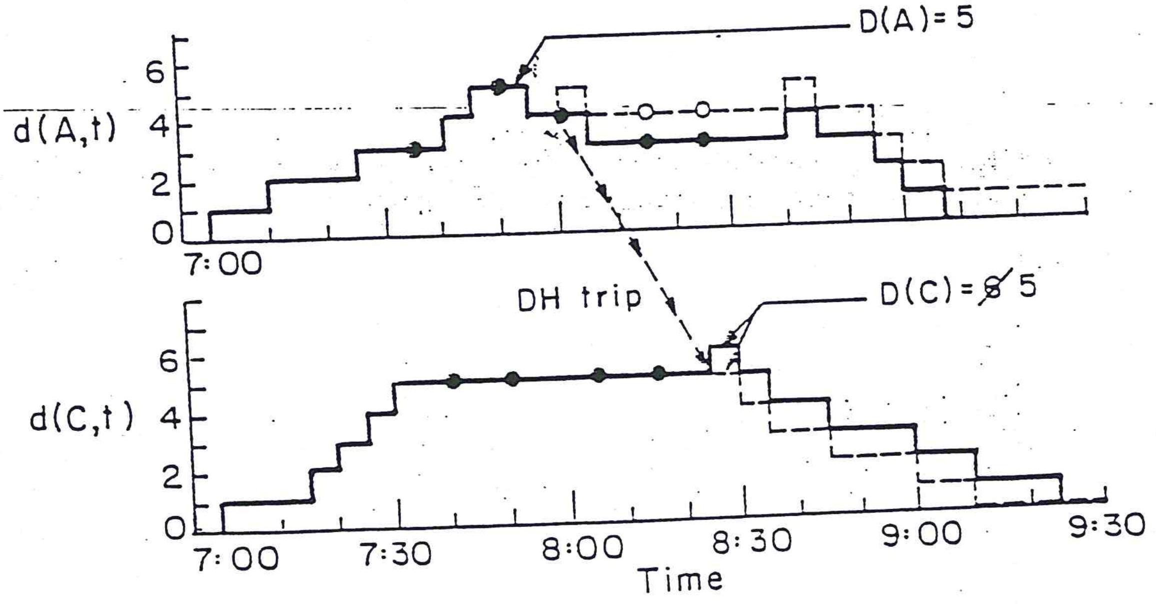


Figure1 : An example of the complete timetable T for which 10 vehicles are required (based o the graphical deficit function method)

3. A PROCEDURE TO EXCLUDE DEPARTURE TIMES: MINIMAX-H ALGORITHM

The basic information required to consider short-turns is the load profile along the entire route. This data is available at most bus properties world-wide and called ride check information (loads and running times along the entire route). Based on this load profile information, each route segment between two adjacent short-turn points can be treated separately. That is, the required number of trips between the (k-1) and k short-turn points for a given direction and time period is :

$$F_k = \max \left(\frac{P_k}{d} , F_{min} \right) \qquad (2)$$

where P_k is the maximum load observed between the two adjacent short-turn points, d is the desired occupancy (load standard) and F_{min} is the minimum required frequency (the reciprocal of what is known as the policy headway).

The complete timetable in current practice is based on the maximum load, P_m, observed along the entire route in a given time period. If the frequency determined from this max. load is not based on the policy headway, then its formulation is :

$$F_m = \frac{P_m}{d} , \quad P_m = \max_k P_k \qquad (3)$$

The manual procedure done by the scheduler to create short-turn trips is simply to exclude departure times in order to set the frequency at each short-turn point k to F_k instead of F_m. The exclusion of departure times is usually performed without any systematic instructions, with the belief that by doing so, it is possible to reduce the number of vehicles required to carry on the timetable.

The result of excluding certain departure times is that some passengers will have to extend their wait at the short-turn points. In order to minimize this adverse effect, it is possible to set the following (minimax H) criterion :

Delete $F_m - F_k$ departure times at k with the objective to minimize the maximum headway to be obtained

It is known that in a deterministic passenger arrival pattern, the wait time is half the headway. Therefore, the above criterion attempts to achieve the minimization of maximum wait. This criterion is called minimax H, and it may represent an adequate passenger level-of-service whenever the scheduler's strategy allows for elimination of some departure times.

In order to solve the optimization problem with the minimax H criterion, a theory was developed in (2). It is based on: (i) representation of the problem on a directed network with a special pattern; (ii) applying a modified shortest-path algorithm (to that described in 15) on the network to determine the minimax headway; and (iii) applying an algorithm to ensure that the exact number of required departures will be included in the optimal solution.

4. OPTIMAL EXTENSION OF THE DETERMINED SHORT-TURN TRIPS

Referring to the example problem in Fig. 1. After the deletion of departures at timepoints A and B in directions A → C and C → A based on the minimax-H algorithm (2), it is possible to construct the new timetable along with the deficit functions, but in this time, at all the three timepoints: A, B, C. That is, in the modified timetable, some trips are initiated at B and some terminate at B in directions A →C and C →A, respectively. Hence, point B becomes also an end/start point and the deficit function description can be applied to it. The new timetable and deficit functions are presented in Fig. 2. Based on the deficit function approach, it is possible to insert a single DH trip from C to B to arrive before or at 8:35 (the beginning of the d(B,t) maximal interval). This results in a minimum fleet size of $\bar{N}_{min} = 9$ vehicles - a saving of one vehicle in contrast to that required for the timetable in Fig. 1. The timetable in Fig. 2 is characterized by the maximum determined short-turn trips for minimizing the fleet size. Following is a method to

Direction	A - C			C - A		
Time-point	A	B	C	C	B	A
Time-table T1**	7:00	7:15	7:40	7:00	7:20	7:35
	7:10	7:25	7:50	7:15	7:35*	-
	7:25	7:40	8:05	7:20	7:40	7:55
	-	7:50*	8:15	7:25	7:45*	-
	-	7:55*	8:20	7:30	7:50*	-
	7:45	8:05	8:30	7:40	8:00	8:15
	-	8:10*	8:35	7:50	8:10	8:25
	8:00	8:20	8:45	8:05	8:30	8:45
	-	8:35*	9:00	8:15	8:40	8:55
	8:25	8:45	9:10	8:20	8:45*	-
	8:40	9:00	9:25	8:27	8:52	9:07

* Departures at B (direction A - C), and arrivals at B (direchtions C - A)
** with DH trip from C to B (8:30 - 8:35)

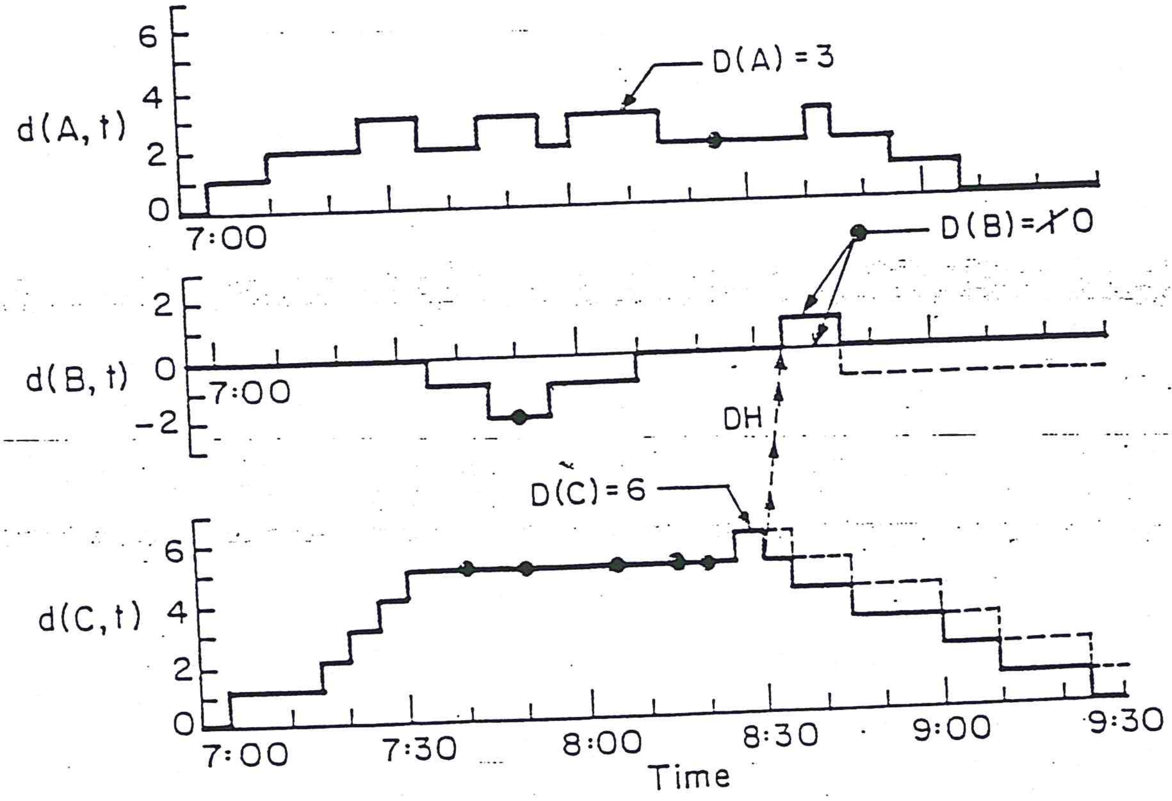


Figure 2:The new timetable T1 with maximum excluded departure times and the three associated deficit functions

reduce (minimize) the number of short-turn trips provided that $\bar{N}_{min}$ is maintained.

4.1. Extensions of Deadheading Trips

Let us denote the modified timetable with maximum short turns by T', the route end points by r_i, i=1,2, and the intermediate short-turn points (belonging to the set R) by U_j, $U_j \in R$, j=1,2, ..., V where there are V short-turn points. The overall schedule to carry on T' might be comprised also from DH trips in order to attain $\bar{N}_{min}$. This overall schedule is designated S. The deficit function properties can be exploited to check if a DH trip can be interpreted as an extension of a short-turn trip in T'.

Based on the deficit function theory, a DH trip can be inserted in a certain time window in order to reduce the fleet size by one. To simplify this possibility, we insert a DH trip from one terminal to terminal k so that its arrival time always coincides with the first time in which d(k,t) attains its maximum. The complete algorithm for this check appears in (1).

4.2. Extensions at Intermediate Short-Turn Points

Let us denote by T'_1 the updated timetable T' including the extensions of DH trips. This T'_1 is now subjected to further extensions at each $U_j \in R$. An extension of a short-turn trip can be viewed as stretching the trip toward the route end points, r_i, i=1,2. An extension does not necessarily mean that the short-turn trip is converted to a full trip along the entire route since it can only be extended partially. That is, an extension can be performed from U_j to U_k (U_j and $U_k \in R$). The extensions at $U \in R$ can be analyzed and executed at three stages: (a) zeroing the maximum deficit function; (b) stretching the maximal interval; and (c) treating the deficit function

hollows. The complete algorithms for these stages appear in (1).

Figure 3 illustrates an example of five extensions on the deficit function $d(k,t)$. The first two -- numbered (1) and (2) refer to the first stage and induce $D(k)$ to decrease from two to zero. Each extension in Fig. 3 refers to a different case, while $d(k,t)$ is updated in sequence. The maximal interval of $d(k,t)$ is indicated by its boundaries t_s and t_e.

The stretching of the maximal interval stage is demonstrated by cases (c), (d) and (e) in Fig. 3. In each case, t_e is updated and in case (f) the procedure stops when the $d(k,t)$'s maximal interval coincides with the span of the schedule horizon. At the third stage, a search is made to determine more extensions at $U_j \in R$ regarding departures and arrivals in hollows. Each hollow in $d(U_j,t)$ contains the same number of arrivals as the number of departures. The procedure developed does not treat hollows which consist of only one point. In Fig. 3 - case (f), for example, there are two hollows: the first consists of two arrivals followed by two departures, and the second -- by a single arrival and departure.

The deficit function theory (11) enables us to construct an extension-search procedure with T_3 which denotes the updated timetable after the first two stages. Finally, if according to the third stage, a new DH trip is introduced, then the procedure of zeroing the maximum deficit function needs to be repeated.

The minimax-H method (2) was applied to the example problem described in Fig. 1. The resultant timetable (with maximum short-turns) appears in the upper part of Fig. 4. The deficit functions of this timetable show that 10 vehicles are required to carry on the timetable without DH trips and 9 vehicles -- with a single DH trip from $d(C,t)$ to arrive to $d(B,t)$ at 8:35. This is shown explicitly by Fig. 2 with $N_{min} = 9$.

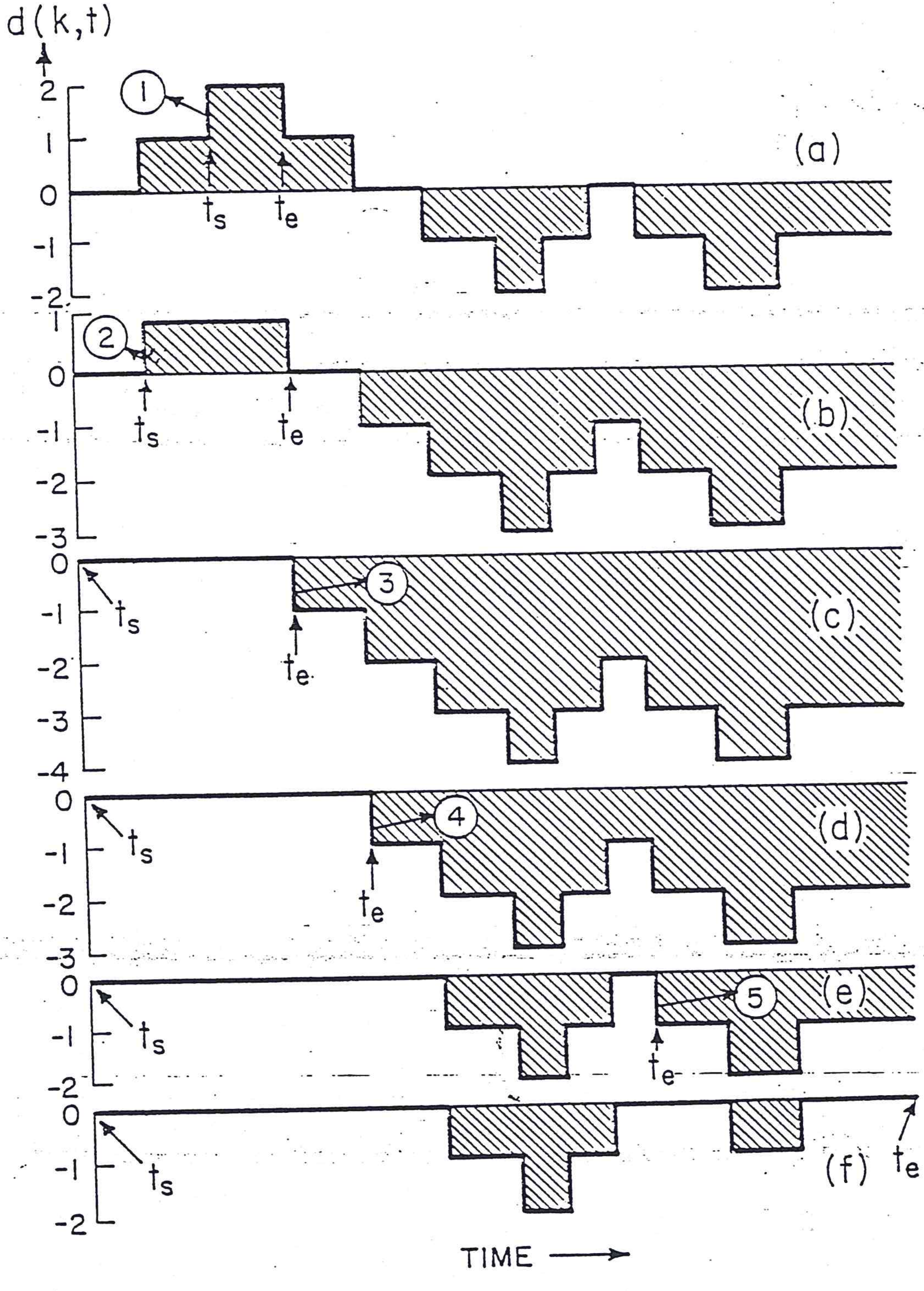


Figure 3: Updated deficit functions at an intermediate short-turn point $U_j = k$ after each of the five indicated extensions

Max. load points

Direction	A - C			C - A		
Time-point	*A*	$\boxed{B}$	*C*	$\boxed{C}$	*B*	*A*
Time-table T_1 with maximum short-turns**	7:00	7:15	7:40	7:00	7:20	7:35
	7:10	7:25	7:50	7:15	7:35*	-
	7:25	7:40	8:05	7:20	7:40	7:55
	-	7:50*	8:15	7:25	7:45*	-
	-	7:55*	8:20	7:30	7:50*	-
	7:45	8:05	8:30	7:40	8:00	8:15
	-	8:10*	8:35	7:50	8:10	8:25
	8:00	8:20	8:45	8:05	8:30	8:45
	-	8:35*	9:00	8:15	8:40	8:55
	8:25	8:45	9:10	8:20	8:45*	-
	8:40	9:00	9:25	8:27	8:52	9:07

* Departures at *B* (direction *A - C*) and arrivals at *B* (direction *C - A*)

** with a *DH* trip from *C* to *B* (8:30 - 8:35)

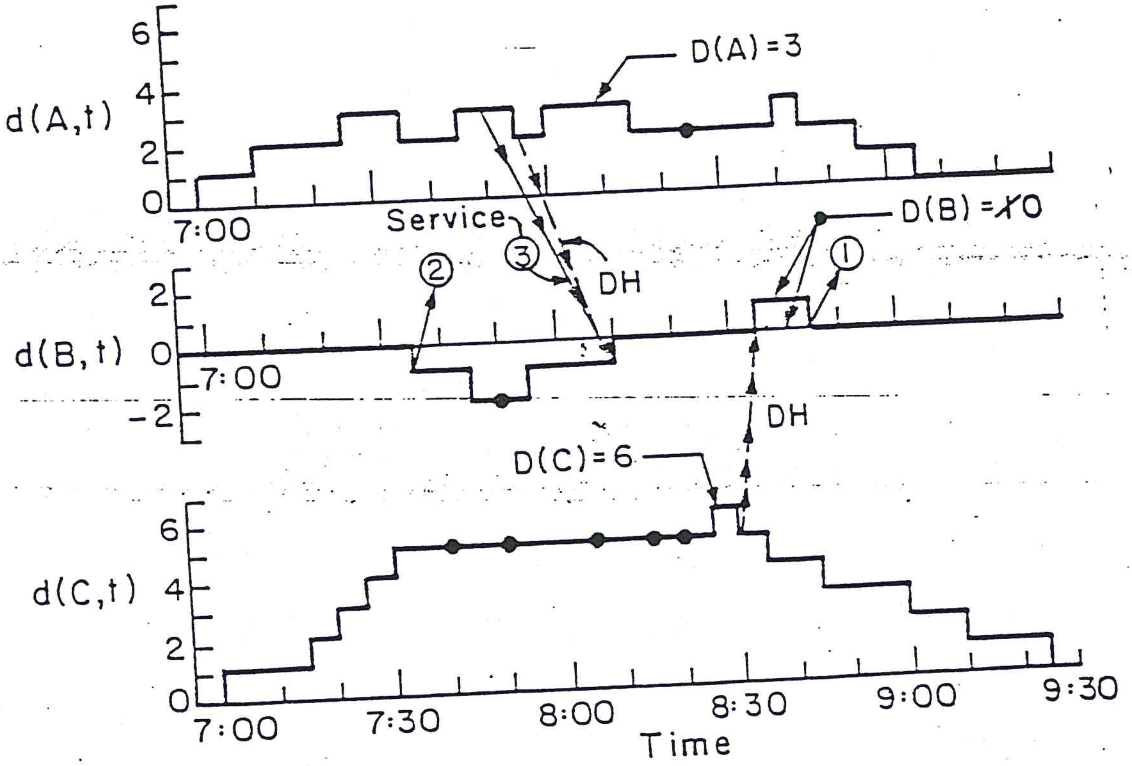


Figure 4: The modified timetable and deficit functions following the minimax H algorithm along with indication of three short-turn trip extensions

service trip. This examination reveals that it cannot be performed and hence, the DH trip remains in the schedule.

Subsequent to this first attempt, the procedures mentioned in section 4.2. are applied. Since D(B) = 0 the algorithm in stage (a) cannot be utilized; however, due to the algorithm in stage (b), extension (1) can be performed (see Figs. 4 and 5). Then, the algorithm in stage (c) is used. One can observe that extension (2) alone affects D(B) to increase by one at 8:10, and the URDHC procedure, therefore, searches for a DH trip that can arrive to B at 8:10. Such a DH trip is inserted from A while ensuring that D(A) remains 3.

The final step is to check the new inserted DH trip with the procedure of section 4.1. This enables us to perform extension (3). Consequently, among the 8 short-turn trips in the timetable of Fig. 4, three were extended to their original schedule while N_{min} remains 9. In other words, the procedures developed identify the minimum (crucial) allowed short-turn trips which are required to reduce fleet size. Figure 5 illustrates the updated deficit functions after the three extensions from which it is possible to observe that no more extensions can be made.

5. CONCLUDING REMARKS

The final product of this work is a set of PL/1 computer programs which execute all the components and tasks of the study.

The outcome of this work can generally be presented in light of the three objectives set forth in section 1. The procedures developed provide the approach and methods to determine the minimum fleet size required to carry on a given schedule.

The second objective of this study is to reduce the number of departures at each short-turn point to that required from a passenger load standpoint, while attempting to minimize its adverse effect on the passenger level-of-service. This objective is fulfilled by adopting the minimax headway criterion, or in other words, minimization of the maximum passenger wait time. Finally, the procedures mentioned in section 4 allow for an additional improvement of the passenger level-of-

Direction	A - C			C - A		
Time-point	*A*	*B*	*C*	*C*	*B*	*A*
Time-table *T*2* with minimum short-turns but same number of vehicles	7:00	7:15	7:40	7:00	7:20	7:35
	7:10	7:25	7:50	7:15	7:35	(7:50)
	7:25	7:40	8:05	7:20	7:40	7:55
	-	7:50	8:15	7:25	7:45	-
	-	7:55	8:20	7:30	7:50	-
	7:45	8:05	8:30	7:40	8:00	8:15
	(7:50)	8:10	8:35	7:50	8:10	8:25
	8:00	8:20	8:45	8:05	8:30	8:45
	-	8:35	9:00	8:15	8:40	8:55
	8:25	8:45	9:10	8:20	8:45	(9:00)
	8:40	9:00	9:25	8:27	8:52	9:07

(9:00) extension① (7:50) ext.②- arrival, (7:50) ext.③- departure

* with a *DH* trip from *C* to *B* (8:30 - 8:35)

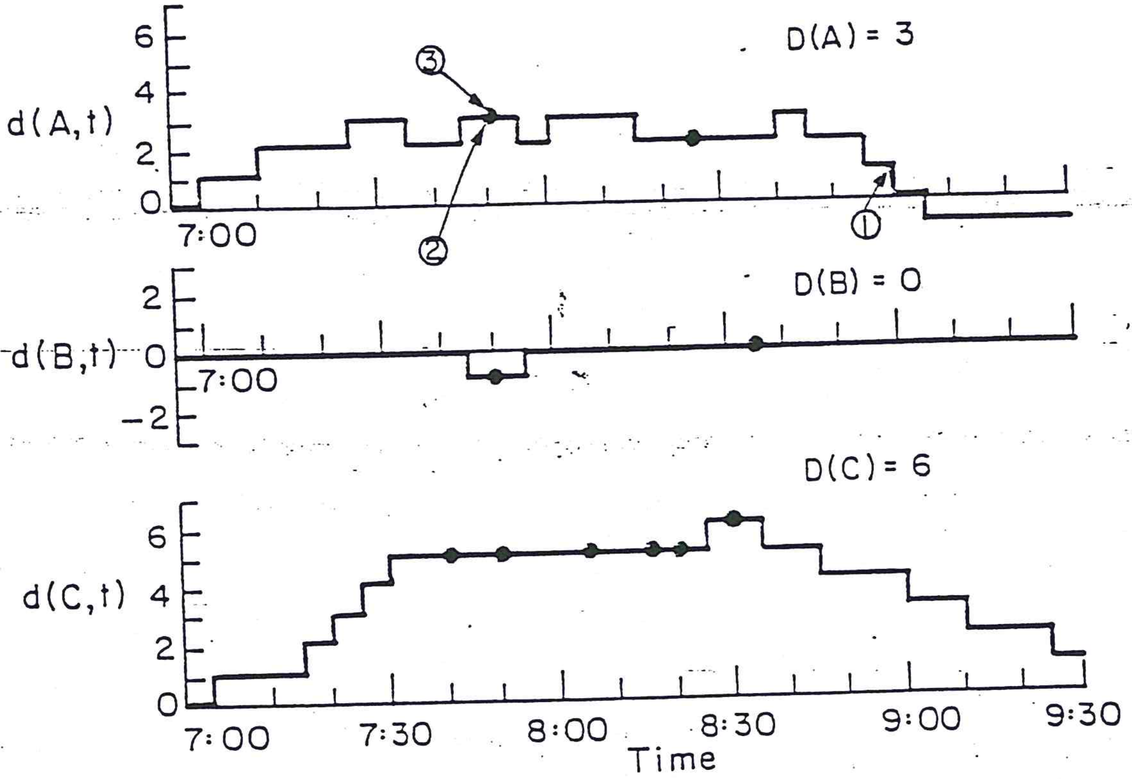


Figure 5: The derived timetable T2 and deficit functions following the three indicated extensions

words, minimization of the maximum passenger wait time. Finally, the procedures mentioned in section 4 allow for an additional improvement of the passenger level-of-service while preserving the minimum fleet size obtained through the elimination of some departure times. These section 4 procedures fulfil the fourth objective of this study.

Future work can be concentrated along the following lines:

(a) Extension of the methods to handle also Origin-Destination (O-D) data whenever it is available, where part of this work is described in (16).

(b) The inclusion of more than two end route points. That is, a transit route may consist of branches, and the procedures developed can easily be extended to consider such cases.

(c) Modification of the procedures to handle a network of interlining routes (in which a vehicle can traverse from one route to another in its block). We notice that when interlining routes are allowed, the minimum fleet size can be further reduced in comparison to the operation of independent routes.

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REFERENCES

1. Ceder, A. (1989), Optimal Design of Transit Sort-Turn Trips." To appear in Transportation Research Record (TRB).

2. Ceder, A. (1989), "A Procedure to Adjust Transit Trip Departure Times Through Minimizing the Maximum Headway." Submitted for publication in Computers & Operations Research - an International Journal.

3. Ceder, A. (1984), "Bus Frequency Determination Using Passenger Count Data." Transportation Research, 18A, pp. 439-453.

4. Ceder, A. (1986), "Methods for Creating Bus Timetables." Transportation Research, 21A, pp. 59-83.

5. Furth, P.G., Brian, D., & Attanucci, J. (1984), "Bus Route and Service Design: An Overview of Strategies for Major Radial Bus Routes." Multisystems, Final Report prepared for U.S. Department of Transportation.

6. Furth, P.G. (1983), "Bus Route & Service Design: Application of Methods & Procedures." Multisystems, Draft Report prepared for U.S. Department of Transportation.

7. Bartlett, T.E. (1957), An Algorithm for the Minimum Number of Transport Units to Maintain a Fixed Schedule." Naval Research Logistics Quarterly, 4, pp. 139-149.

8. Gertsbach, I. & Gurevich, Y. (1977), "Constructing an Optimal Fleet for a Transportation Schedule." Transportation Science, 11, pp. 20-36.

9. Salzborn, F.J.M. (1972), "Optimum Bus Scheduling." Transportation Science, 6, pp. 137-148.

10. Salzborn, F.J.M. (1972), "A Note on the Fleet Routing Models for Transportation Systems." Transportation Science, 6, pp. 335-337.

11. Ceder, A. & Stern, H.I. (1981), "Deficit Function Bus Scheduling with Deadheading Trip Insertions for Fleet Size Reduction." Transportation Science, 15, pp. 338-363.

12. Stern, H.I. & Ceder, A. (1983), "An Improved Lower Bound to the Minimum Fleet Size Problem." Transportation Science, 17, pp. 471-477.

13. Ceder, A. & Stern, H.I. (1982), "Graphical Person-Machine Interactive Approach for Bus Scheduling." Transportation Research Record, 857, pp. 69-72.

14. Ceder, A. & Stern, H.I. (1985), "The Variable Trip Procedures Used in the AUTOBUS Vehicle Scheduler." Computer Scheduling of Public Transport 2, J.M. Rousseau (Ed.), North-Holland Publishing Company, pp. 371-390.

15. Dijkstra, E.W. (1959), "A Note on Two Problems in Connection with Graphs." Numerische Mathematik, 1, pp. 269-271.

16. Ceder, A. (1988), "Designing Transit Short-Turn Trips with the Elimination of Imbalanced Loads." Computer-Aided Transit Scheduling, Springer-Verlag Publication, pp. 212-225.